# Learning Structured Predictors 

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https://dmetrics.com

## Outline

Part IIntroductionFour Approaches to Sequence PredictionGreedy Sequence Prediction
Part II
Factored Sequence Prediction
Algorithms for Factored ModelsLog-linear Factored Models
Part III
Structured Perceptron
Log-linear Models and CRFs
Dependency Parsing
Summary and Conclusion

## Supervised (Structured) Prediction

- Learning to predict: given training data

$$
\left\{\left(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}\right),\left(\mathbf{x}^{(2)}, \mathbf{y}^{(2)}\right), \ldots,\left(\mathbf{x}^{(m)}, \mathbf{y}^{(m)}\right)\right\}
$$

learn a predictor $\mathrm{x} \rightarrow \mathrm{y}$ that works well on unseen inputs x

- Non-Structured Prediction: outputs y are atomic
- Binary classification: $\mathbf{y} \in\{-1,+1\}$
- Multiclass classification: $\mathbf{y} \in\{1,2, \ldots, L\}$
- Structured Prediction: outputs y are structured
- Sequence prediction: y are sequences
- Parsing: y are trees
- ...


## Named Entity Recognition

| $\mathbf{y}$ | PER | - | QNT | - | - | ORG | ORG | - | TIME |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{x}$ | Jim | bought | 300 | shares | of | Acme | Corp. | in | 2006 |

## Named Entity Recognition

```
y PER - QNT - - ORG ORG - TIME
x Jim bought 300 shares of Acme Corp. in 2006
    y PER PER - - LOC
    x Jack London went to Paris
y 
    y y PER 
```


## Part-of-speech Tagging

| $\mathbf{y}$ | NOUN | NOUN | VERB | NOUN |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{x}$ | Fruit | flies | like | bananas |

## Syntactic Dependency Parsing



## Machine Translation

| 'Ce n'est pas |
| :--- | :--- |
| un autre |
| problème de |
| classification.' |$\quad$| 'This is not |
| :--- |
| another |
| classification |
| problem.' |

(illustration by Ben Taskar)
x are sentences in some source language (e.g. French) $y$ are sentence translations in a target language (e.g. English)

## Object Detection


(Kumar and Hebert, 2003)
x are images
y are grids labeled with object types

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## Today's Goals

- Introduce basic concepts for structured prediction
- We will focus on sequence prediction
- What can we can borrow from standard classification?
- Learning paradigms and algorithms, in essence, work here too
- However, computations behind algorithms are prohibitive
- Today's main topics:
- Transition systems versus factored models
- Feature representations of structured input-output pairs
- Prediction algorithms
- Learning algorithms: Perceptron and CRF
- Local and global learning losses
- Topics not covered:
- NLP task overviews, evaluation, state-of-the-art systems
- Hidden (structured) representations
- Unsupervised learning (induction of labeled sequences and trees)


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## Sequence Prediction

| $\mathbf{y}$ | PER | PER | - | - | LOC |
| :--- | :--- | :---: | :---: | :---: | :---: |
| $\mathbf{x}$ | Jack | London | went | to | Paris |

## Sequence Prediction

- $\mathbf{x}=x_{1} x_{2} \ldots x_{n}$ are input sequences, $x_{i} \in \mathcal{X}$
- $\mathbf{y}=y_{1} y_{2} \ldots y_{n}$ are output sequences, $y_{i} \in\{1, \ldots, L\}$
- Goal: given training data

$$
\left\{\left(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}\right),\left(\mathbf{x}^{(2)}, \mathbf{y}^{(2)}\right), \ldots,\left(\mathbf{x}^{(m)}, \mathbf{y}^{(m)}\right)\right\}
$$

learn a predictor $\mathrm{x} \rightarrow \mathrm{y}$ that works well on unseen inputs x

- What is the form of our prediction model?


## Exponentially-many Solutions

- Let $\mathcal{Y}=\{-$, PER, LOC $\}$
- The solution space (all output sequences):

| Jack | London | went | to | Paris |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

- Each path is a possible solution
- For an input sequence of size $n$, there are $|\mathcal{Y}|^{n}$ possible outputs


## Exponentially-many Solutions

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Jack London went to Paris

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## Approach 1：Label Classifiers

| PER | PER | - | - |
| :---: | :---: | :---: | :---: |
| Jack | London | went | to |

－Scoring of individual labels at each position

$$
\text { 对单个位置 } \mathrm{t} \quad \hat{y}_{t}=\underset{l \in\{\mathrm{LOC}, \text { PER, }-\}}{\operatorname{argmax}} \operatorname{score}(\mathrm{x}, t, l)
$$

－For linear models， $\operatorname{score}(\mathbf{x}, t, l)=\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, t, l)$
－ $\mathbf{f}(\mathbf{x}, t, l) \in \mathbb{R}^{d}$ represents an assignment of label $l$ for $x_{t}$
－ $\mathrm{w} \in \mathbb{R}^{d}$ is a vector of parameters（learned），has a weight for each feature in f
－Can capture interactions between full input x and one output label l
e．g．：current word，surrounding words，capitalization，prefix－suffix，gazetteer，．．．
－Can not capture interactions between output labels！

## Approach 2：Transition－based Sequence Prediction


－Score one label at a time，left－to－right，conditioning on previous predictions：

$$
\text { 对单个位置 } \mathrm{t} \quad \hat{y}_{t}=\underset{l \in\{\mathrm{LOC}, \text { PER, }-\}}{\operatorname{argmax}} \operatorname{score}\left(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1: t-1}\right)
$$

－Captures interactions between full input x and prefixes of the output sequence
－Greedy predictions，prone to search errors even with beam search
－Why left－to－right and not right－to－left？

## Approach 3：Factored Sequence Prediction


－Scoring of label bigrams（pairs of adjacent labels）at each position：
对整个序列y

$$
\hat{\mathbf{y}}=\underset{\substack{\operatorname{rangmax}}}{\operatorname{argmar}} \operatorname{score}(\mathbf{x}, \mathbf{y})=\underset{y}{\operatorname{argmax}} \sum_{\mathcal{Y}^{n}}^{\arg } \sum_{i=1}^{n} \operatorname{score}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
$$

－Output sequence factored into label bigrams
－Captures interactions between full input x and factors of output sequence
－Prediction is exact for many types of factorizations

## Approach 4：Re－Ranking

| PER | PER | － | － | LOC |
| :---: | :---: | :---: | :---: | :---: |
| PER | LOC | － | － | LOC |
| LOC | LOC | － | － | LOC |
| PER | PER | － | － | PER |
| PER | PER | PER | － | LOC |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| Jack | London | went | to | Paris |

对整个序列 $y \quad \hat{\mathbf{y}}=\underset{\mathbf{y} \in \mathcal{A}\left(\mathcal{Y}^{n}\right)}{\operatorname{argmax}} \operatorname{score}(\mathbf{x}, \mathbf{y})$
－Scoring of full inputs and outputs：very expressive！
－Relies on an active set $\mathcal{A}\left(\mathcal{Y}^{n}\right)$ of full outputs，enumerated exhaustively
－A base model is used to select active set
－The base model follows one of the previous approaches

## Sequence Prediction: Summary of Approaches

|  | input-output representation | exact prediction? |
| :---: | :---: | :---: |
| label classifiers | only individual labels | yes |
| transition-based | full history of decisons | $\begin{gathered} \text { no } \\ \text { (greedy, beam search) } \end{gathered}$ |
| factored | label factors | yes |
| re-ranking | full | limited to active set |
| take home | 1: expressivity-tr | ty trade-off |

## Sequence Prediction: Summary of Approaches

|  | input-output <br> representation | exact prediction? |
| :---: | :---: | :---: |
| label classifiers | only individual labels | yes |
| transition-based | full history of decisons | (greedy, beam search) |
| re-ranking | label factors | yes |

take home message 1: expressivity-tractability trade-off take home message 2: always pick the simplest approach that suits the task at hand

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## Greedy Sequence Prediction



- Run a greedy classifier left-to-right:
- For $t=1 \ldots n$ :

$$
\hat{y}_{t}=\underset{l \in\{\text { LOC, PER, }-\}}{\operatorname{argmax}} \operatorname{score}\left(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1: t-1}\right)
$$

- What is the form of $\operatorname{score}\left(\mathbf{x}, t, l, \hat{y}_{1: t-1}\right)$ ?
- We focus on linear scoring functions: $\operatorname{score}\left(\mathbf{x}, t, l, \hat{y}_{1: t-1}\right)=\mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, t, l, \hat{y}_{1: t-1}\right)$


## Representations in Greedy Sequence Prediction

- In linear greedy sequence prediction, at time $t$

$$
\operatorname{score}\left(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1: t-1}\right)=\mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1: t-1}\right)
$$

- $\mathrm{w} \in \mathbb{R}^{d}$ is a parameter vector, to be learned
- $\mathbf{f}\left(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1: t-1}\right) \in \mathbb{R}^{d}$ is a feature vector
- Goal: guess the correct $l$ at position $t$
- How to construct $\mathrm{f}\left(\mathrm{x}, t, l, \hat{\mathbf{y}}_{1: t-1}\right)$ ?
- New trend: representation learning
- Old school: manually with feature templates


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## Indicator Features for One Label Only

- $\mathbf{f}(\mathbf{x}, t, l)$ is a vector of $d$ features representing label $l$ for $x_{t}$
- What's in a feature $\mathbf{f}_{j}(\mathbf{x}, t, l)$ ?
- Anything we can compute using x and $t$ and $l$
- Anything that indicates whether $l$ is (not) a good label for $x_{t}$
- Indicator features: binary-valued features looking at:
- a simple pattern of x and target position $t$
- and the candidate label $l$ for position $t$

$$
\begin{aligned}
& \mathbf{f}_{j}(\mathbf{x}, t, l)= \begin{cases}1 & \text { if } x_{t}=\text { London and } l=\mathrm{LOC} \\
0 & \text { otherwise }\end{cases} \\
& \mathbf{f}_{k}(\mathbf{x}, t, l)= \begin{cases}1 & \text { if } x_{t+1}=\text { went and } l=\mathrm{LOC} \\
0 & \text { otherwise }\end{cases}
\end{aligned}
$$

- Indicator features produce sparse feature vectors


## Feature Templates

- Feature templates generate many indicator features
- A feature template is identified by a type, and a number of values
- Example: template WORD indicates the current word

$$
\mathbf{f}_{\langle\text {word }, a, w\rangle}(\mathbf{x}, t, l)= \begin{cases}1 & \text { if } x_{t}=w \text { and } l=a \\ 0 & \text { otherwise }\end{cases}
$$

- A feature of this type is identified by the tuple 〈WORD, $a, w$,
- Generates a feature for every label $a \in \mathcal{Y}$ and every word $w$
- Feature vectors and weight vectors are indexed by feature tuples



## Feature Templates

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$$

－A feature of this type is identified by the tuple $\langle$ word，$a, w\rangle$
－Generates a feature for every label $a \in \mathcal{Y}$ and every word $w$
－Feature vectors and weight vectors are indexed by feature tuples
－In feature－based models：
－Define feature templates manually
－Instantiate the templates on every set of values in the training data $\rightarrow$ generates a very high－dimensional feature space
－Define parameter vector w indexed by such feature tuples

|  |  |
| :---: | :---: |
| $\mathrm{a}=\mathrm{PER} \quad\langle\mathrm{WORD}, \mathrm{PER}, \mathrm{I}\rangle$〈WORD，PER，you〉〈WORD，PER，went〉〈WORD，PER，saw〉〈WORD，PER，John〉〈WORD，PER，Marie〉〈WORD，PER，London〉〈WORD，PER，Paris |  |
| $\mathrm{a}=\mathrm{LOC} \begin{gathered}\text { 〈WORORD，LOC，I〉 }\rangle \\ \text { 〈WORD，LOC，you〉 }\end{gathered}$〈WORD，LOC，went〉〈WORD，LOC，saw〉〈WORD，LOC，John〉〈WORD，LOC，Marie〉 （WORD，LOC，London） （WORD，LOC，Paris） |  |

－Let the learning algorithm choose the relevant features

## More Features for NE Recognition

In practice, construct $\mathrm{f}(\mathrm{x}, t, l)$ by

- Define a number of simple patterns of $\mathbf{x}$ and $t$
- current word $x_{t}$
- is $x_{t}$ capitalized?
- next word
- $x_{t}$ has digits?
- prefixes/suffixes of size $1,2,3, \ldots$
- is $x_{t}$ a known location?
- is $x_{t}$ a known person?
- previous word
- current and next words together
- other combinations
- Define feature templates by combining patterns with labels $l$
- Generate actual features by instantiating templates on training data

| - | Current word |
| :---: | :---: |
|  | Caps, digits |
|  | Prefixes, suffixes |
|  | Next word |
|  | Previous word |
| PER | Current word |
|  | Caps, digits |
|  | Prefixes, suffixes |
|  | Next word |
|  | Previous word |
| LOC | Current word |
|  | Caps, digits |
|  | Prefixes, suffixes |
|  | Next word |
|  | Previous word |

## Feature Templates in Greedy Sequence Prediction

| $\mathbf{y}$ | PER | PER | - |  |
| :--- | :--- | :---: | :---: | :---: |
| $\mathbf{x}$ | Jack | London | went | to Paris |

- $\mathbf{f}\left(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1: t-1}\right)$ has access to all preceding labels
- Example: A template for word + current label + previous label:

$$
\mathbf{f}_{\langle\mathrm{wB}, a, b, w\rangle}\left(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1: t-1}\right)=\left\{\begin{array}{cc}
1 & \text { if } x_{t}=w \text { and } \\
& \begin{array}{c}
\hat{\mathbf{y}}_{t-1}=a \text { and } l=b \\
0
\end{array} \\
\text { otherwise }
\end{array}\right.
$$

- In practice:
- Preceeding labels next to $t$
- Bag-of-labels in $\hat{\mathbf{y}}_{1: t-1}$
- Combinations with other features
- Neural networks automatically induce "good" features out of x and $\hat{\mathbf{y}}_{1: t-1}$


## Transition Systems (general form)

- Given an input x , a transition system defines:
- A set of states $\mathcal{S}(\mathrm{x})$
- An initial state $s_{0} \in \mathcal{S}(\mathbf{x})$, and a set of final states $S_{\infty} \subseteq \mathcal{S}(\mathbf{x})$
- A set of allowed actions $\mathcal{A}(s, \mathbf{x})$ for all $s \in \mathcal{S}(\mathbf{x})$
- A transition function transition : $s \times a \rightarrow s^{\prime}$
- A scoring function: score : $\mathbf{x} \times s \times a \rightarrow \mathbb{R}$
- To predict output y from input x :
- $s=s_{0}$
- while $s \notin S_{\infty}$ :
- $a=\operatorname{argmax}_{a \in \mathcal{A}(s, \mathbf{x})} \operatorname{score}(\mathbf{x}, s, a)$
- $s=\operatorname{transition}(s, a)$
- extract y from $s$
- Simple, very fast and expressive! Very popular in NLP:
- Greedy sequence prediction (one label at a time, left-to-right or right-to-left)
- Shift-reduce parsing (more later)
- Word segmentation, machine translation, ...


## Greedy Predictions are not Optimal, even with Beam Search



- Greedy sequence predictions can not undo decisions at a later stage
- Sometimes the model is right at a global scope, but not at each greedy step!
- Solution: Beam Search
- General local search method
- Maintains several good hypotheses, instead of just the best one
- Many strategies, sometimes specific to the task and transition system
- Empirically, it often improves over greedy search


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## Factored Sequence Predictors



$$
\hat{\mathbf{y}}=\underset{\mathbf{y} \in \mathcal{Y}^{n}}{\operatorname{argmax}} \sum_{i=1}^{n} \operatorname{score}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
$$

Next questions:
-What is the form of $\operatorname{score}(\mathbf{x}, i, a, b)$ ?
We will use linear scoring functions: $\operatorname{score}(\mathbf{x}, i, a, b)=\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, a, b)$

- There are exponentially-many sequences y for a given x , how do we solve the argmax problem?


## Representations Factored at Bigrams

| y: | PER | PER | - | - | LOC |
| :--- | :--- | :--- | :---: | :---: | :---: |
| x: | Jack | London went | to | Paris |  |

- $\operatorname{score}(\mathbf{x}, i, a, b)=\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, a, b)$
- $\mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)$
- A $d$-dimensional feature vector of a label bigram at $i$
- Each dimension is typically a boolean indicator (0 or 1 )
- $\mathbf{f}(\mathbf{x}, \mathbf{y})=\sum_{i=1}^{n} \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)$
- A d-dimensional feature vector of the entire y
- Aggregated representation by summing bigram feature vectors
- Each dimension is now a count of a feature pattern


## Representations Factored at Bigrams

| y: | PER | PER | - | - | LOC |
| :--- | :--- | :--- | :---: | :---: | :---: |
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- $\mathbf{f}(\mathbf{x}, \mathbf{y})=\sum_{i=1}^{n} \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)$
- A $d$-dimensional feature vector of the entire $\mathbf{y}$
- Aggregated representation by summing bigram feature vectors
- Each dimension is now a count of a feature pattern


## Linear Factored Sequence Prediction

$$
\underset{\mathbf{y} \in \mathcal{Y}^{n}}{\operatorname{argmax}} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y}) \quad \text { where } \quad \mathbf{f}(\mathbf{x}, \mathbf{y})=\sum_{i=1}^{n} \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
$$

- Note the linearity of the expression:

$$
\begin{aligned}
\operatorname{score}(\mathbf{x}, \mathbf{y}) & =\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y}) \\
& =\mathbf{w} \cdot \sum_{i=1}^{n} \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right) \\
& =\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right) \\
& =\sum_{i=1}^{n} \operatorname{score}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
\end{aligned}
$$

## Predicting with Factored Sequence Models

- Assume we have a score function $\operatorname{score}(\mathbf{x}, i, a, b)$
- Given $\mathrm{x}_{1: n}$ find:

$$
\underset{\mathbf{y} \in \mathcal{Y}^{n}}{\operatorname{argmax}} \sum_{i=1}^{n} \operatorname{score}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
$$

- Use the Viterbi algorithm, takes $O\left(n|\mathcal{Y}|^{2}\right)$
- Notational change: since $\mathrm{x}_{1: n}$ is fixed we will use

$$
s(i, a, b)=\operatorname{score}(\mathbf{x}, i, a, b)
$$

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## Viterbi for Factored Sequence Models

- Given scores $s(i, a, b)$ for each position $i$ and output bigram $a, b$, find:

$$
\underset{\mathbf{y} \in \mathcal{Y}^{n}}{\operatorname{argmax}} \sum_{i=1}^{n} s\left(i, y_{i-1}, y_{i}\right)
$$

- Intuition: consider this example x and two alternative solutions y and $\mathrm{y}^{\prime}$ :

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{x}$ | Jack | London | went | to | Paris | before | visiting | Lisbon |
| $\mathbf{y}$ | PER | LOC | - | - | LOC | - | - | LOC |
| $\mathbf{y}^{\prime}$ | PER | PER | - | - | LOC | - | - | LOC |

- What is the score of $y^{\prime}$ relative to the score of $y$ ?

$$
s\left(\mathbf{x}, \mathbf{y}^{\prime}\right)=s(\mathbf{x}, \mathbf{y})+
$$

- 
- 


## Viterbi for Factored Sequence Models

- Given scores $s(i, a, b)$ for each position $i$ and output bigram $a, b$, find:

$$
\underset{\mathbf{y} \in \mathcal{Y}^{n}}{\operatorname{argmax}} \sum_{i=1}^{n} s\left(i, y_{i-1}, y_{i}\right)
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| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{x}$ | Jack | London | went | to | Paris | before | visiting | Lisbon |
| $\mathbf{y}$ | PER | LOC | - | - | LOC | - | - | LOC |
| $\mathbf{y}^{\prime}$ | PER | PER | - | - | LOC | - | - | LOC |

- What is the score of $y^{\prime}$ relative to the score of $y$ ?

$$
\begin{aligned}
s\left(\mathbf{x}, \mathbf{y}^{\prime}\right)=s(\mathbf{x}, \mathbf{y}) & +s(2, \mathrm{PER}, \mathrm{PER})-s(2, \mathrm{PER}, \mathrm{LOC}) \\
& +s(3, \mathrm{PER},-)-s(3, \mathrm{LOC},-)
\end{aligned}
$$

output sequences that share bigrams also share their scores

## Viterbi recurrence

- Viterbi is a dynamic programming algorithm that uses the following recurrence
- Assume that, for a certain position $i$ and each label $l \in \mathcal{Y}$, we have the best sub-sequence from positions 1 to $i$ ending with label $l$ :

$$
\begin{array}{cccc}
1 & \cdots & i & i+1
\end{array}
$$

best subsequence with $y_{i}=$ PER
best subsequence with $y_{i}=$ LOC
best subsequence with $y_{i}=-$

- What is the best sequence up to position $i+1$ with $y_{i+1}=$ LOC?


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best subsequence with $y_{i}=\mathrm{PER}$
best subsequence with $y_{i}=$ LOC
best subsequence with $y_{i}=-$

- What is the best sequence up to position $i+1$ with $y_{i+1}=$ LOC?


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- Assume that, for a certain position $i$ and each label $l \in \mathcal{Y}$, we have the best sub-sequence from positions 1 to $i$ ending with label $l$ :

- What is the best sequence up to position $i+1$ with $y_{i+1}=$ LOC?


## Viterbi for Factored Sequence Models

$$
\hat{\mathbf{y}}=\operatorname{argmax}_{\mathbf{y} \in \mathcal{Y}^{n}} \sum_{i=1}^{n} s\left(i, y_{i-1}, y_{i}\right)
$$

- Definition: score of optimal sequence for $\mathrm{x}_{1: i}$ ending with $a \in \mathcal{Y}$

$$
\delta(i, a)=\max _{\mathbf{y} \in \mathcal{Y}^{i}: y_{i}=a} \sum_{j=1}^{i} s\left(j, y_{j-1}, y_{j}\right)
$$

- Use the following recursions, for all $a \in \mathcal{Y}$, for $i=2 \ldots n$ :

$$
\begin{aligned}
\delta(1, a) & =s\left(1, y_{0}=\text { NULL }, a\right) \\
\delta(i, a) & =\max _{b \in \mathcal{Y}} \delta(i-1, b)+s(i, b, a)
\end{aligned}
$$

- The optimal score for x is $\max _{a \in \mathcal{Y}} \delta(n, a)$
- The optimal sequence $\hat{\mathbf{y}}$ can be recovered through back-pointers
- Cost: $O\left(n|\mathcal{Y}|^{2}\right)$


## Viterbi for Factored Sequence Models

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\end{aligned}
$$

- The optimal score for x is $\max _{a \in \mathcal{Y}} \delta(n, a)$
- The optimal sequence $\hat{y}$ can be recovered through back-pointers
- Homework: rewrite the Viterbi equations such that the algorithm proceeds right-to-left. Observe that the factored model remains the same (i.e. it is not a directional model)


## Variations of Viterbi

- Sparse Viterbi
- Only a few labels in $\mathcal{Y}$ apply to a position
- Only a few label bigrams are possible
- A sparse implementation cuts the $O\left(|\mathcal{Y}|^{2}\right)$ factor
- Higher-order Viterbi: factorize at trigrams instead of bigrams
- Cost $O\left(n|\mathcal{Y}|^{3}\right)$
- Very common in POS tagging (using sparse Viterbi to cut the $O\left(|\mathcal{Y}|^{3}\right)$ cost factor)
- $k$-best Viterbi: return the best $k$ sequences (not just the single best)
- Used in re-ranking approaches and some loss functions
- Forward-Backward: Viterbi for sum-product computations (instead of max-sum)


## Forward-Backward Max-Sum Computations

- The Viterbi algorithm solves a max-sum recurrence

$$
\max _{\mathbf{y} \in \mathcal{Y}^{n}} \sum_{i=1}^{n} s\left(i, y_{i-1}, y_{i}\right)
$$

- The sum-product recurrence is also very useful (more later)

$$
\sum_{\mathbf{y} \in \mathcal{Y}^{n}} \prod_{i=1}^{n} s\left(i, y_{i-1}, y_{i}\right)
$$

- The same style of dynamic programming works


## Forward Algorithm

$$
\sum_{\mathbf{y} \in \mathcal{Y}^{n}} \prod_{i=1}^{n} s\left(i, y_{i-1}, y_{i}\right)
$$

- Definition: forward quantities
$1 \quad \alpha(i, a) \quad a^{i+1}{ }^{n}$

$$
\alpha(i, a)=\sum_{\mathbf{y}_{1: i} \in \mathcal{Y}^{i}: y_{i}=a} \prod_{j=1}^{i} s\left(j, y_{j-1}, y_{j}\right)
$$

- Use the following recursions, for all $a \in \mathcal{Y}$, for $i=2 \ldots n$ :

$$
\begin{aligned}
& \alpha(i, a)=s\left(1, y_{0}=\text { NULL }, a\right) \\
& \alpha(i, a)=\sum_{b \in \mathcal{Y}} \alpha(i-1, b) * s(i, b, a)
\end{aligned}
$$

- The total sum-product is $\sum_{a} \alpha(n, a)$
- Like Viterbi, the forward algorithm runs in $O\left(n|\mathcal{Y}|^{2}\right)$


## Backward Algorithm

$$
\sum_{\mathbf{y} \in \mathcal{Y}^{n}} \prod_{i=1}^{n} s\left(i, y_{i-1}, y_{i}\right)
$$

- Definition: backward quantities

$$
\beta(i, a)=\sum_{\mathbf{y}_{i: n} \in \mathcal{Y}(n-i+1): y_{i}=a} \prod_{j=i+1}^{n} s\left(j, y_{j-1}, y_{j}\right)
$$

- Now the recursions run backwards! For all $a \in \mathcal{Y}$, for $i=n-1 \ldots 1$ :

$$
\begin{aligned}
\beta(n, a) & =1 \\
\beta(i, a) & =\sum_{b \in \mathcal{Y}} s(i, a, b) * \beta(i+1, b)
\end{aligned}
$$

- The total sum-product is $\sum_{a} s\left(1, y_{0}=\right.$ NULL, $\left.a\right) * \beta(1, a)$
- Like Viterbi and forward algorithms, the backward algorithm runs in $O\left(n|\mathcal{Y}|^{2}\right)$


## Outline

Part IIntroductionFour Approaches to Sequence PredictionGreedy Sequence Prediction
Part II
Factored Sequence Prediction
Algorithms for Factored ModelsLog-linear Factored Models
Part III
Structured Perceptron
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## Log-linear Models for Sequence Prediction

- Model the conditional distribution:

$$
\operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w})=\frac{\exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x} ; \mathbf{w})}
$$

where

- $\mathrm{f}(\mathrm{x}, \mathrm{y})$ represents x and y with $d$ features
- $\mathrm{w} \in \mathbb{R}^{d}$ are the parameters of the model
- $Z(\mathbf{x} ; \mathbf{w})$ is a normalizer called the partition function

$$
Z(\mathbf{x} ; \mathbf{w})=\sum_{\mathbf{z} \in \mathcal{Y}^{*}} \exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})\}
$$

- To predict the best sequence

$$
\underset{\mathbf{y} \in \mathcal{Y}^{n}}{\operatorname{argmax}} \operatorname{Pr}(\mathbf{y} \mid \mathbf{x})
$$

## Log-linear Models: Name

- Let's take the log of the conditional probability:

$$
\begin{aligned}
\log \operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w}) & =\log \frac{\exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x} ; \mathbf{w})} \\
& =\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})-\log \sum_{y} \exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\} \\
& =\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})-\log Z(\mathbf{x} ; \mathbf{w})
\end{aligned}
$$

- Partition function: $Z(\mathbf{x} ; \mathbf{w})=\sum_{\mathbf{z}} \exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})\}$
- $\log Z(\mathbf{x} ; \mathbf{w})$ is a constant for a fixed $\mathbf{x}$
- In the log space, computations are linear,
i.e., we model log-probabilities using a linear predictor


## Making Predictions with Log-Linear Models

- For tractability, assume $\mathbf{f}(\mathbf{x}, \mathrm{y})$ decomposes into bigrams:

$$
\mathbf{f}\left(\mathbf{x}_{1: n}, \mathbf{y}_{1: n}\right)=\sum_{i=1}^{n} \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
$$

- Given $\mathbf{w}$, given $\mathrm{x}_{1: n}$, find:

$$
\begin{aligned}
\underset{\mathbf{y}_{1: n}}{\operatorname{argmax}} \operatorname{Pr}\left(\mathbf{y}_{1: n} \mid \mathbf{x}_{1: n} ; \mathbf{w}\right) & =\underset{\mathbf{y}}{\operatorname{amax}} \frac{\exp \left\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)\right\}}{Z(\mathbf{x} ; \mathbf{w})} \\
& =\underset{\mathbf{y}}{\operatorname{amax}} \exp \left\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)\right\} \\
& =\underset{\mathbf{y}}{\operatorname{amax}} \sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
\end{aligned}
$$

- We can use the Viterbi algorithm


## Making Predictions with Log-Linear Models

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& =\underset{\mathbf{y}}{\operatorname{amax}} \sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
\end{aligned}
$$

- We can use the Viterbi algorithm


## Probability of an Output Sequence given an Input Sequence

- Given $\mathbf{x}$ and $\mathbf{y}$, compute $\operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w})=\frac{\exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x} ; \mathbf{w})}$
- To compute $Z(\mathbf{x} ; \mathbf{w})$ we need to sum over $\mathcal{Y}^{n}$ !
- But with some algebraic massaging: (let $\left.s\left(i, y_{i-1}, y_{i}\right)=\mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)\right)$

$$
\begin{aligned}
Z(\mathbf{x} ; \mathbf{w}) & =\sum_{\mathbf{y}} \exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\} \\
& =\sum_{\mathbf{y}} \exp \left\{\sum_{i=1}^{n} s\left(i, y_{i-1}, y_{i}\right)\right\} \\
& =\sum_{\mathbf{y}} \prod_{i=1}^{n} \exp \left\{s\left(i, y_{i-1}, y_{i}\right)\right\}
\end{aligned}
$$

- $Z(\mathbf{x} ; \mathbf{w})$ is a sum-product computation: forward algorithm (with exponentiated scores)!
- $Z(\mathbf{x} ; \mathbf{w})=\sum_{a} \alpha(n, a)$


## Marginal Probability of a Single Label

$$
\begin{aligned}
& \text { PER } \\
& \text { Paris Jackson }
\end{aligned}
$$

$$
i
$$

- What's the probability that token $i$ has label $a$ ?
- We need to compute the marginal distribution of $y_{i}$ :

$$
\mu_{i}(a)=\operatorname{Pr}\left(y_{i}=a \mid \mathbf{x} ; \mathbf{w}\right)=\sum_{\mathbf{y} \in \mathcal{Y}^{n}: y_{i}=a} \operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w})
$$

- Use forward-backward (using exponentiated scores)
- Recall that $Z(\mathbf{x} ; \mathbf{w})=\sum_{l} \alpha(n, l)$


## Marginal Probability of a Single Label

|  | $\alpha(i, \mathrm{PER})$ | PER | $\beta(i, \mathrm{PER})$ |  |
| :---: | :---: | :---: | :---: | :---: |
| I saw | Paris | Jackson | playing |  |
|  | $i$ |  |  |  |

- What's the probability that token $i$ has label $a$ ?
- We need to compute the marginal distribution of $y_{i}$ :

$$
\begin{aligned}
\mu_{i}(a)=\operatorname{Pr}\left(y_{i}=a \mid \mathbf{x} ; \mathbf{w}\right) & =\sum_{\mathbf{y} \in \mathcal{Y}^{n}: y_{i}=a} \operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w}) \\
& =\text { (algebraic massaging }) \\
& =\frac{\alpha(i, a) * \beta(i, a)}{Z(\mathbf{x} ; \mathbf{w})}
\end{aligned}
$$

- Use forward-backward (using exponentiated scores)
- Recall that $Z(\mathbf{x} ; \mathbf{w})=\sum_{l} \alpha(n, l)$


## Marginal Probability of a Label Bigram

I saw | PER | PER |  |  |
| :---: | :---: | :---: | :---: |
|  | Paris | Jackson | playing |
|  | $i-1$ | $i$ |  |

- What's the probability that token $i-1$ has label $a$ and token $i$ has label $b$ ?
- We need to compute the marginal distribution of label bigrams at position $i$ :

$$
\mu_{i}(a, b)=\operatorname{Pr}\left(y_{i-1}=a, y_{i}=b \mid \mathbf{x} ; \mathbf{w}\right)=\sum_{\mathbf{y} \in \mathcal{Y}^{n}: y_{i-1}=a, y_{i}=b} \operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w})
$$

- Again forward-backward (using exponentiated scores)
- Recall that $Z(\mathrm{x}: \mathbf{w})=\sum, \alpha(n . l)$


## Marginal Probability of a Label Bigram

|  | $\exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, \mathrm{PER}, \mathrm{PER})\}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\alpha(i-1, \mathrm{PER})$ | PER | PER | $\beta(i, \mathrm{PER})$ |  |
| I | saw | Paris | Jackson | playing |  |
|  |  | $i-1$ | $i$ |  |  |
|  |  |  |  |  |  |

- What's the probability that token $i-1$ has label $a$ and token $i$ has label $b$ ?
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$$
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\mu_{i}(a, b)=\operatorname{Pr}\left(y_{i-1}=a, y_{i}=b \mid \mathbf{x} ; \mathbf{w}\right) & =\sum_{\mathbf{y} \in \mathcal{Y}^{n}: y_{i-1}=a, y_{i}=b} \operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w}) \\
& =(\text { algebraic massaging }) \\
& =\frac{\alpha(i-1, a) * \exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, a, b)\} * \beta(i, b)}{Z(\mathbf{x} ; \mathbf{w})}
\end{aligned}
$$

- Again forward-backward (using exponentiated scores)
- Recall that $Z(\mathbf{x} ; \mathbf{w})=\sum_{l} \alpha(n, l)$


## Linear Factored Sequence Prediction

$$
\underset{\mathbf{y} \in \mathcal{Y}^{n}}{\operatorname{argmax}} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})
$$

- Factored representation, e.g. based on bigrams

$$
\mathbf{f}(\mathbf{x}, \mathbf{y})=\sum_{i=1}^{n} \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
$$

- Flexible, arbitrary features of full x and the factors
- Efficient prediction using Viterbi
- In probabilistic models, efficient computation of marginals using Forward-Backward
- Next, learning w:
- The Structured Perceptron
- Probabilistic log-linear models:
- Local learning, a.k.a. Maximum-Entropy Markov Models
- Global learning, a.k.a. Conditional Random Fields


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## The Structured Perceptron

## Collins（2002）

－Set $\mathbf{w}=\mathbf{0}$
－For $t=1 \ldots T$
－For each training example（ $\mathbf{x}, \mathbf{y}$ ）
1．Compute $\mathbf{z}=\operatorname{argmax}_{\mathbf{z}} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})$
2．If $\mathbf{z} \neq \mathbf{y}$

$$
\mathbf{w} \leftarrow \mathbf{w}+\mathbf{f}(\mathbf{x}, \mathbf{y})-\mathbf{f}(\mathbf{x}, \mathbf{z})
$$

－Return w
增加已知正确标签y 的 w
减少 错误预测标签 z 的 w
注意 f 是指示函数，取值 $0 / 1$

## The Structured Perceptron + Averaging

Freund and Schapire (1999); Collins (2002)

- Set $\mathbf{w}=\mathbf{0}, \mathbf{w}^{a}=\mathbf{0}$
- For $t=1 \ldots T$
- For each training example ( $\mathbf{x}, \mathbf{y}$ )

1. Compute $\mathbf{z}=\operatorname{argmax}_{\mathbf{z}} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})$
2. If $\mathbf{z} \neq \mathbf{y}$

$$
\mathbf{w} \leftarrow \mathbf{w}+\mathbf{f}(\mathbf{x}, \mathbf{y})-\mathbf{f}(\mathbf{x}, \mathbf{z})
$$

3. $\mathrm{w}^{\mathrm{a}}=\mathrm{w}^{\mathrm{a}}+\mathrm{w}$

- Return $\mathrm{w}^{\mathrm{a}}$


## Perceptron Updates: Example

| $\mathbf{y}$ | PER | PER | - | - | LOC |
| :--- | :--- | :--- | :---: | :---: | :---: |
| $\mathbf{z}$ | PER | LOC | - | - | LOC |
| $\mathbf{x}$ | Jack | London | went | to | Paris |

- Let y be the correct output for x .
- Say we predict z instead, under our current w
- The update is:

$$
\begin{aligned}
\mathbf{g} & =\mathbf{f}(\mathbf{x}, \mathbf{y})-\mathbf{f}(\mathbf{x}, \mathbf{z}) \\
& =\sum_{i} \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)-\sum_{i} \mathbf{f}\left(\mathbf{x}, i, z_{i-1}, z_{i}\right) \\
& =\mathbf{f}(\mathbf{x}, 2, \text { PER, PER })-\mathbf{f}(\mathbf{x}, 2, \text { PER, LOC }) \\
& +\mathbf{f}(\mathbf{x}, 3, \text { PER, }-)-\mathbf{f}(\mathbf{x}, 3, \text { LOC },-)
\end{aligned}
$$

- Perceptron updates are typically very sparse


## Properties of the Perceptron

- Online algorithm. Often much more efficient than "batch" algorithms
- If the data is separable, it will converge to parameter values with 0 errors
- Number of errors before convergence is related to a definition of margin. Can also relate margin to generalization properties
- In practice:

1. Averaging improves performance a lot
2. Typically reaches a good solution after only a few (say 5) iterations over the training set
3. Often performs nearly as well as CRFs, or SVMs

- Structured Perceptron and Beam Search:
- Transition systems can not recover the argmax solution
- Structured Perceptron can use beam search instead (i.e. an approximation to argmax)
- See Collins and Roark (2004); Zhang and Clark (2011); Huang et al. (2012)


## Averaged Perceptron Convergence

| Iteration | Accuracy |
| :---: | :---: |
| 1 | 90.79 |
| 2 | 91.20 |
| 3 | 91.32 |
| 4 | 91.47 |
| 5 | 91.58 |
| 6 | 91.78 |
| 7 | 91.76 |
| 8 | 91.82 |
| 9 | 91.88 |
| 10 | 91.91 |
| 11 | 91.92 |
| 12 | 91.96 |
| $\ldots$ |  |



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## Log-linear Models for Sequence Prediction

- Model the conditional distribution:

$$
\operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w})=\frac{\exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x} ; \mathbf{w})}
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where

- $\mathrm{f}(\mathrm{x}, \mathrm{y})$ represents x and y with $d$ features
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- $Z(\mathbf{x} ; \mathbf{w})$ is a normalizer called the partition function

$$
Z(\mathbf{x} ; \mathbf{w})=\sum_{\mathbf{z} \in \mathcal{Y}^{*}} \exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})\}
$$

- To predict the best sequence

$$
\underset{\mathbf{y} \in \mathcal{Y}^{n}}{\operatorname{argmax}} \operatorname{Pr}(\mathbf{y} \mid \mathbf{x})
$$

## Parameter Estimation in Log-Linear Models

$$
\operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w})=\frac{\exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x} ; \mathbf{w})}
$$

- Given training data

$$
\left\{\left(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}\right),\left(\mathbf{x}^{(2)}, \mathbf{y}^{(2)}\right), \ldots,\left(\mathbf{x}^{(m)}, \mathbf{y}^{(m)}\right)\right\}
$$

- How to estimate w?
- Define the conditional log-likelihood (or cross-entropy) of the data:
- $L(\mathbf{w})$ measures how well w explains the data. A good value for w will give a high value for $\operatorname{Pr}\left(\mathbf{y}^{(k)} \mid \mathbf{x}^{(k)} ; \mathbf{w}\right)$ for all $k=1$
- We want w that maximizes $L(w)$


## Parameter Estimation in Log-Linear Models

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$$

- How to estimate w?
- Define the conditional log-likelihood (or cross-entropy) of the data:

$$
L(\mathbf{w})=\sum_{k=1}^{m} \log \operatorname{Pr}\left(\mathbf{y}^{(k)} \mid \mathbf{x}^{(k)} ; \mathbf{w}\right)
$$

- $L(\mathbf{w})$ measures how well $\mathbf{w}$ explains the data. A good value for $\mathbf{w}$ will give a high value for $\operatorname{Pr}\left(\mathbf{y}^{(k)} \mid \mathbf{x}^{(k)} ; \mathbf{w}\right)$ for all $k=1 \ldots m$.
- We want $\mathbf{w}$ that maximizes $L(\mathbf{w})$


## Learning Log-Linear Models: Loss + Regularization

- Solve:

$$
\mathbf{w}^{*}=\underset{\mathbf{w} \in \mathbb{R}^{d}}{\operatorname{argmin}} \overbrace{-L(\mathbf{w})}^{\text {Loss }}+\overbrace{\frac{\lambda}{2}\|\mathbf{w}\|^{2}}^{\text {Regularization }}
$$

where

- The first term is the negative conditional log-likelihood
- The second term is a regularization term, it penalizes solutions with large norm
- $\lambda \in \mathbb{R}$ controls the trade-off between loss and regularization
- Convex optimization problem $\rightarrow$ gradient descent
- Two common losses based on log-likelihood that make learning tractable:


## Learning Log-Linear Models: Loss + Regularization

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- $\lambda \in \mathbb{R}$ controls the trade-off between loss and regularization
- Convex optimization problem $\rightarrow$ gradient descent
- Two common losses based on log-likelihood that make learning tractable:
- Local Loss (MEMM): assume that $\operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w})$ decomposes
- Global Loss (CRF): assume that $\mathrm{f}(\mathrm{x}, \mathrm{y})$ decomposes

Local Log-linear Loss (a.k.a. Maximum Entropy Markov Models) McCallum, Freitag, and Pereira (2000)

- If we apply the chain rule:

$$
\begin{aligned}
\operatorname{Pr}\left(\mathbf{y}_{1: n} \mid \mathbf{x}_{1: n}\right) & =\operatorname{Pr}\left(y_{1} \mid \mathbf{x}_{1: n}\right) \times \operatorname{Pr}\left(\mathbf{y}_{2: n} \mid \mathbf{x}_{1: n}, y_{1}\right) \\
& =\operatorname{Pr}\left(y_{1} \mid \mathbf{x}_{1: n}\right) \times \prod_{i=2}^{n} \operatorname{Pr}\left(y_{i} \mid \mathbf{x}_{1: n}, \mathbf{y}_{1: i-1}\right)
\end{aligned}
$$

- Markov assumption (the model becomes factored):

$$
\operatorname{Pr}\left(y_{i} \mid \mathbf{x}_{1: n}, \mathbf{y}_{1: i-1}\right)=\operatorname{Pr}\left(y_{i} \mid \mathbf{x}_{1: n}, y_{i-1}\right)
$$

- Now we can write

$$
\operatorname{Pr}\left(\mathbf{y}_{1: n} \mid \mathbf{x}_{1: n}\right)=\operatorname{Pr}\left(y_{1} \mid \mathbf{x}_{1: n}\right) \times \prod_{i=2}^{n} \operatorname{Pr}\left(y_{i} \mid \mathbf{x}_{1: n}, \mathbf{y}_{i-1}\right)
$$

## Parameter Estimation with Local Log-Linear Markov Models

$$
\operatorname{Pr}\left(y_{1: n} \mid \mathbf{x}_{1: n}\right)=\operatorname{Pr}\left(y_{1} \mid \mathbf{x}_{1: n}\right) \times \prod_{i=2}^{n} \operatorname{Pr}\left(y_{i} \mid \mathbf{x}_{1: n}, i, y_{i-1}\right)
$$

- The log-linear model is normalized locally (i.e. at each position):

$$
\operatorname{Pr}\left(y \mid \mathbf{x}, i, y^{\prime}\right)=\frac{\exp \left\{\mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, y^{\prime}, y\right)\right\}}{Z\left(\mathbf{x}, i, y^{\prime}\right)}
$$

- The log-likelihood is also local :

$$
\begin{gathered}
L(\mathbf{w})=\sum_{k=1}^{m} \sum_{i=1}^{n^{(k)}} \log \operatorname{Pr}\left(\mathbf{y}_{i}^{(k)} \mid \mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)}\right) \\
\frac{\partial L(\mathbf{w})}{\partial \mathbf{w}_{j}}=\frac{1}{m} \sum_{k=1}^{m} \sum_{i=1}^{n^{(k)}}[\overbrace{\mathbf{f}_{j}\left(\mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)}, \mathbf{y}_{i}^{(k)}\right)}^{\text {observed }}-\overbrace{\sum_{y \in \mathcal{Y}} \operatorname{Pr}\left(\mathbf{y} \mid \mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)}, y\right) \mathbf{f}_{j}\left(\mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)}, y\right)}^{\text {expected }}
\end{gathered}
$$

## Conditional Random Fields

## Lafferty, McCallum, and Pereira (2001)

- Log-linear model of the conditional distribution:

$$
\operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w})=\frac{\exp \{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x})}
$$

where

- x and y are input and output sequences
- $f(x, y)$ is a feature vector of $x$ and $y$ that decomposes into factors
- w are model parameters
- To predict the best sequence

$$
\hat{\mathbf{y}}=\underset{\mathbf{y} \in \mathcal{Y}^{*}}{\operatorname{argmax}} \operatorname{Pr}(\mathbf{y} \mid \mathbf{x})
$$

- Log-Likelihood at the global (sequence) level:

$$
L(\mathbf{w})=\sum_{k=1}^{m} \log \operatorname{Pr}\left(\mathbf{y}^{(k)} \mid \mathbf{x}^{(k)} ; \mathbf{w}\right)
$$

## Computing the Gradient in CRFs

Consider a parameter $\mathbf{w}_{j}$ and its associated feature $\mathbf{f}_{j}$ :

$$
\frac{\partial L(\mathbf{w})}{\partial \mathbf{w}_{j}}=\frac{1}{m} \sum_{k=1}^{m}[\overbrace{\mathbf{f}_{j}\left(\mathbf{x}^{(k)}, \mathbf{y}^{(k)}\right)}^{\text {observed }}-\overbrace{\sum_{\mathbf{y} \in \mathcal{Y}^{*}} \operatorname{Pr}\left(\mathbf{y} \mid \mathbf{x}^{(k)} ; \mathbf{w}\right) \mathbf{f}_{j}\left(\mathbf{x}^{(k)}, \mathbf{y}\right)}^{\text {expected }}]
$$

where

$$
\mathbf{f}_{j}(\mathbf{x}, \mathbf{y})=\sum_{i=1}^{n} \mathbf{f}_{j}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
$$

- First term: observed value of $\mathbf{f}_{j}$ in training examples
- Second term: expected value of $\mathrm{f}_{j}$ under current w they require summing over all sequences $\mathbf{y} \in \mathcal{Y}^{n}$


## Computing the Gradient in CRFs

- For an example $\left(\mathbf{x}^{(k)}, \mathbf{y}^{(k)}\right)$ :

$$
\sum_{\mathbf{y} \in \mathcal{Y}^{n}} \operatorname{Pr}\left(\mathbf{y} \mid \mathbf{x}^{(k)} ; \mathbf{w}\right) \sum_{i=1}^{n} \mathbf{f}_{j}\left(\mathbf{x}^{(k)}, i, y_{i-1}, y_{i}\right)=\sum_{i=1}^{n} \sum_{a, b \in \mathcal{Y}} \mu_{i}^{k}(a, b) \mathbf{f}_{j}\left(\mathbf{x}^{(k)}, i, a, b\right)
$$

- $\mu_{i}^{k}(a, b)$ is the marginal probability of having labels $(a, b)$ at position $i$ :

$$
\mu_{i}^{k}(a, b)=\operatorname{Pr}\left(\langle i, a, b\rangle \mid \mathbf{x}^{(k)} ; \mathbf{w}\right)=\sum_{\mathbf{y} \in \mathcal{Y}^{n}}: y_{i-1}=a, y_{i}=b
$$

- The quantities $\mu_{i}^{k}$ can be computed efficiently in $O\left(n L^{2}\right)$ using the forward-backward algorithm


## CRFs: summary so far

- Log-linear models for sequence prediction, $\operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w})$
- Computations factorize on label bigrams
- Model form:

$$
\underset{\mathbf{y} \in \mathcal{Y}^{*}}{\operatorname{argmax}} \sum_{i} \mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
$$

- Prediction: uses Viterbi
- Parameter estimation:
- Gradient-based methods, in practice L-BFGS or SGD
- Computation of gradient uses forward-backward


## CRFs: summary so far

- Log-linear models for sequence prediction, $\operatorname{Pr}(\mathbf{y} \mid \mathbf{x} ; \mathbf{w})$
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$$

- Prediction: uses Viterbi
- Parameter estimation:
- Gradient-based methods, in practice L-BFGS or SGD
- Computation of gradient uses forward-backward
- Next Question: Local or Global loss?


## Local vs. Global Log-linear Losses

Local Loss: $\quad \operatorname{Pr}(\mathbf{y} \mid \mathbf{x})=\prod_{i=1}^{n} \frac{\exp \left\{\mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)\right\}}{Z\left(\mathbf{x}, i, y_{i-1} ; \mathbf{w}\right)}$

CRFs: $\quad \operatorname{Pr}(\mathbf{y} \mid \mathbf{x})=\frac{\exp \left\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)\right\}}{Z(\mathbf{x})}$

- Both exploit the same factorization, i.e. same features
- Same computations to compute $\operatorname{argmax}_{\mathbf{y}} \operatorname{Pr}(\mathbf{y} \mid \mathbf{x})$
- Local loss is locally normalized; CRFs globally normalized
- Local loss assumes that $\operatorname{Pr}\left(y_{i} \mid x_{1: n}, y_{1: i-1}\right)=\operatorname{Pr}\left(y_{i} \mid x_{1: n}, y_{i-1}\right)$
- Leads to "Label Bias Problem" (Lafferty et al., 2001; Andor et al., 2016)
- Local loss is cheaper to train (reduces to multiclass MaxEnt learning)
- CRFs are easier to extend to other structures


## Learning Structure Predictors: summary so far

- Linear models for sequence prediction

$$
\underset{\mathbf{y} \in \mathcal{Y}^{*}}{\operatorname{argmax}} \sum_{i} \mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
$$

- Computations factorize on label bigrams
- Decoding: using Viterbi
- Marginals: using forward-backward
- Parameter estimation:
- Perceptron, Log-likelihood, SVMs
- Extensions from classification to the structured case
- Optimization methods:
- Stochastic (sub)gradient methods (LeCun et al., 1998; Shalev-Shwartz et al., 2011)
- Exponentiated Gradient (Collins et al., 2008)
- SVM Struct (Tsochantaridis et al., 2005)
- Structured MIRA (Crammer et al., 2005)


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## Dependency Parsing



## Dependency Parsing



## Theories of Syntactic Structure

## Dependency Trees



## Constituent Trees



- Main element: constituents (or phrases, or bracketings)
- Constituents $=$ abstract linguistic units
- Results in nested trees


## Dependency Parsing: Arc-factored models

McDonald, Pereira, Ribarov, and Hajič (2005)


- Parse trees decompose into single dependencies $\langle h, m\rangle$

$$
\underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \sum_{\langle h, m\rangle \in y} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, h, m)
$$

- Each arc or dependency $(h, m)$ is scored independently of each other
- Some features: $\quad \mathbf{f}_{1}(\mathbf{x}, h, m)=[$ "saw" $\rightarrow$ "movie" $]$

$$
\mathbf{f}_{2}(\mathbf{x}, h, m)=[\text { distance }=+2]
$$

- Tractable inference algorithms exist


## MST Parsing for Arc-factored models

McDonald, Pereira, Ribarov, and Hajič (2005)

- Parsing problem, given a sentenc $x$ :

$$
\underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \sum_{\langle h, m\rangle \in \mathbf{y}} \operatorname{score}(\mathbf{x}, h, m)
$$

- Can be formulated as a directed Maximum Spanning Tree (MST) problem:

- The Chu-Liu-Edmonds algorithm finds the optimal tree in $O\left(n^{2}\right)$


## The Eisner Algorithm for Arc-factored models

Eisner (1996); McDonald and Pereira (2006); Carreras (2007); Koo and Collins (2010)

(illustration by Joakim Nivre)

Extension to higher-order parsing:

horizontal

- First-order $O\left(n^{3}\right)$
- Second-order:
- Horizontal $O\left(n^{3}\right)$ (McDonald and Pereira, 2006)
- Vertical $O\left(n^{4}\right)$ (Carreras, 2007)
- Third-order $O\left(n^{4}\right)$ (Koo and Collins, 2010)


## Transition-based Parsing: Nivre’s Arc-Standard System

Nivre (2008)

- State:
- Buffer: list of upcoming words to be parsed
- Stack: stack of subtrees that are already parsed
- Parsing actions:
- Shift: shift next word in the buffer to the task
- Left-arc ( $l$ ): add a left arc between the two top subtrees of the stack, with label $l$
- Right-arc ( $l$ ): add a right arc between the two top subtrees of the stack, with label $l$
- Parsing is linear in the sentence length, very fast! But prone to greedy mistakes!
- Parsing model: score a candidate action in the context of a state
- Has access to the full sentence and the full history of actions


## Arc-Standard Parsing: Example

## (illustration by Miguel Ballesteros)



| transition | Stack <br> [] | Buffer <br> [Mark, Watney, visited, Mars] |
| :--- | :--- | :--- |
|  |  |  |

Mark Watney visited Mars

## Arc-Standard Parsing: Example

## (illustration by Miguel Ballesteros)



| transition | Stack <br> [] | Buffer <br> [Mark, Watney, visited, Mars] |
| :--- | :--- | :--- |
| SHIFT | $[$ Mark $]$ | [Watney, visited, Mars] |

Mark Watney visited Mars

## Arc-Standard Parsing: Example

## (illustration by Miguel Ballesteros)



| transition | Stack | Buffer |
| :--- | :--- | :--- |
|  | [] | $[$ Mark, Watney, visited, Mars $]$ |
| SHIFT | $[$ Mark $]$ | $[$ Watney, visited, Mars $]$ |
| SHIFT | $[$ Mark, Watney $]$ | $[$ visited, Mars] |

Mark Watney visited Mars

## Arc-Standard Parsing: Example

## (illustration by Miguel Ballesteros)



| transition | Stack | Buffer |
| :--- | :--- | :--- |
|  | [] | $[$ Mark, Watney, visited, Mars] |
| SHIFT | $[$ Mark $]$ | $[$ Watney, visited, Mars] |
| SHIFT | $[$ Mark, Watney $]$ | [visited, Mars] |
| LA(NAME) | [Watney] | [visited, Mars] |

$\overbrace{\text { Mark Watney visited Mars }}^{\text {NAME }}$

## Arc-Standard Parsing: Example

## (illustration by Miguel Ballesteros)



| transition | Stack | Buffer |
| :--- | :--- | :--- |
|  | [] | $[$ Mark, Watney, visited, Mars] |
| SHIFT | $[$ Mark $]$ | $[$ Watney, visited, Mars $]$ |
| SHIFT | $[$ Mark, Watney $]$ | $[$ visited, Mars $]$ |
| LA(NAME) | [Watney $]$ | $[$ visited, Mars $]$ |
| SHIFT | $[$ Watney, visited $]$ | $[$ Mars $]$ |



## Arc-Standard Parsing: Example

## (illustration by Miguel Ballesteros)



| transition | Stack <br> [] | Buffer <br> [Mark, Watney, visited, Mars] |
| :---: | :---: | :---: |
| SHIFT | [Mark] | [Watney, visited, Mars] |
| SHIFT | [Mark, Watney] | [visited, Mars] |
| LA(NAME) | [Watney] | [visited, Mars] |
| SHIFT | [Watney, visited] | [Mars] |
| LA(SUBJ) | [visited] | [Mars] |

## Arc-Standard Parsing: Example

## (illustration by Miguel Ballesteros)



## Arc-Standard Parsing: Example

## (illustration by Miguel Ballesteros)



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## Linear (Structured) Prediction

- Multiclass classification

$$
\underset{\mathbf{y} \in\{1, \ldots, L\}}{\operatorname{argmax}} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})
$$

- Sequence prediction (bigram factorization)

$$
\underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})=\underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \sum_{i} \mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, \mathbf{y}_{i-1}, \mathbf{y}_{i}\right)
$$

- Dependency parsing (arc-factored)

$$
\underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})=\underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \sum_{\langle h, m, l\rangle \in y} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, h, m, l)
$$

- Factored models: Applicable to other tasks and factorizations
- Alternative: transition systems (very fast and expressive, but prone to search errors)


## Factored Sequence Prediction: from Linear to Non-linear

$$
\operatorname{score}(\mathbf{x}, \mathbf{y})=\sum_{i} \mathrm{~s}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)
$$

- Linear:

$$
\mathrm{s}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)=\mathbf{w} \cdot \mathbf{f}\left(\mathbf{x}, i, \mathbf{y}_{i-1}, \mathbf{y}_{i}\right)
$$

- Non-linear, using a feed-forward neural network:

$$
\mathrm{s}\left(\mathbf{x}, i, y_{i-1}, y_{i}\right)=\mathbf{w} \cdot\left[e_{y_{i-1}, y_{i}} \otimes h(\mathbf{f}(\mathbf{x}, i))\right]
$$

where:

$$
h(\mathbf{f}(\mathbf{x}, i))=\sigma\left(W^{2} \sigma\left(W^{1} \sigma\left(W^{0} \mathbf{f}(\mathbf{x}, i)\right)\right)\right)
$$

- Remarks:
- The non-linear model computes a hidden representation of the input
- Still factored: Viterbi and Forward-Backward work
- Parameter estimation becomes non-convex, use backpropagation


## Recurrent Sequence Prediction



- Induction of hidden vectors (i.e. embeddings) that keep track of previous observations and predictions
- Making predictions is not tractable
- In practice: greedy predictions or beam search
- Making predictions was not tractable for transition systems either!
- Learning is non-convex, so what?
- Popular methods: RNN, LSTM, Spectral Models, ...


## Neural Architectures for Named Entity Recognition

Guillaume Lample ${ }^{*}$ Miguel Ballesterosth ${ }^{\text {t/ }}$
Sandeep Subramanian ${ }^{\boldsymbol{\wedge}}$ Kazuya Kawakami ${ }^{\boldsymbol{\wedge}}$ Chris Dyer ${ }^{\boldsymbol{\wedge}}$


| Model | $\mathbf{F}_{\mathbf{1}}$ |
| :--- | :---: |
| Collobert et al. (2011)* | 89.59 |
| Lin and Wu (2009) | 83.78 |
| Lin and Wu (2009)* | 90.90 |
| Huang et al. (2015)* | 90.10 |
| Passos et al. (2014) | 90.05 |
| Passos et al. (2014)* | 90.90 |
| Luo et al. (2015)* + gaz | 89.9 |
| Luo et al. (2015)* + gaz + linking | 91.2 |
| Chiu and Nichols (2015) | 90.69 |
| Chiu and Nichols (2015)* | 90.77 |
| LSTM-CRF (no char) | 90.20 |
| LSTM-CRF | $\mathbf{9 0 . 9 4}$ |
| S-LSTM (no char) | 87.96 |
| S-LSTM | 90.33 |

Table 1: English NER results (CoNLL-2003 test set).

Xuezhe Ma and Eduard Hovy


| Model | POS |  | NER |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Dev | Test | Dev |  |  |  | Test |  |
|  | Acc. | Acc. | Prec. | Recall | F1 | Prec. | Recall | F1 |
|  | 96.56 | 96.76 | 92.04 | 89.13 | 90.56 | 87.05 | 83.88 | 85.44 |
| BLSTM | 96.88 | 96.93 | 92.31 | 90.85 | 91.57 | 87.77 | 86.23 | 87.00 |
| BLSTM-CNN | 97.34 | 97.33 | 92.52 | 93.64 | 93.07 | 88.53 | 90.21 | 89.36 |
| BRNN-CNN-CRF | 97.46 | 97.55 | 94.85 | 94.63 | 94.74 | 91.35 | 91.06 | 91.21 |

Table 3: Performance of our model on both the development and test sets of the two tasks, together with three baseline systems.

| Model | Acc. |
| :--- | :---: |
| Giménez and Màrquez (2004) | 97.16 |
| Toutanova et al. (2003) | 97.27 |
| Manning (2011) | 97.28 |
| Collobert et al. (2011) | $\ddagger$ |
| Santos and Zadrozny (2014) | $\ddagger$ |
| Shen et al. (2007) | 97.29 |
| Sun (2014) | 97.32 |
| Søgaard (2011) | 97.36 |
| This paper | 97.50 |

Table 4: POS tagging accuracy of our model on test data from WSJ proportion of PTB, together

| Model | F1 |
| :--- | :---: |
| Chieu and Ng (2002) | 88.31 |
| Florian et al. (2003) | 88.76 |
| Ando and Zhang (2005) | 89.31 |
| Collobert et al. (2011) | 89.59 |
| Huang et al. (2015) | 99.1 |
| Chiu and Nichols (2015) | 90.10 |
| Ratinov and Roth (2009) | 90.77 |
| Lin and Wu (2009) | 90.80 |
| Passos et al. (2014) | 90.90 |
| Lample et al. (2016) | 90.94 |
| Luo et al. (2015) | 91.20 |
| This paper | $\mathbf{9 1 . 2 1}$ |

Table 5: NER F1 score of our model on test data set from CoNLL-2003. For the purpose of com-

Thanks!

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